Derivatives & Integration

① Derivaties can also be called as Differential Equations.

When $y = f(x)$ $\longrightarrow$ y = Dependent variable
x = Independent variable

example : ① Demand = y
price of commodity = x then
y is a function of x i.e. $y = f(x)$

② speed = y
acceleration = x
Here also y is a function of x
$y = g(x)$
i.e. speed is dependent on acceleration.

② If Demand is the function of price
$\therefore$ with a small change in price, what is its impact on quantity demanded?

$\Downarrow$

You need to find, Derivative of quantity demanded with respect to price.

$$= \frac{d}{dp}(Q) = \frac{dQ}{dp}$$

③ $y = f(x)$

then

with a small change in value of x, its impact on value of y $= f'(x) = \dfrac{dy}{dx}$

$$y = f(x)$$

$$\therefore \frac{dy}{dx} = f'(x) = \text{Differential eq}^n \text{ of } x$$

④ If y is a function of x i.e. $y = f(x)$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{dy}{dx}}$$

⇓

Derivative by First principle

⇓

$$\text{Diff. function of } x = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h - x} = \text{Derivative of } y \text{ with respect of } x$$

Derivative by First principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

⑤

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^1	1
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^5	$5x^4$
x^n	$n.(x)^{n-1}$
x^{12}	$12.x^{11}$
x^{35}	$35.x^{34}$
x^m	$m.(x)^{m-1}$

⑥

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
a^x	$a^x.\log a$
5^x	$5^x.\log_e 5$
8^x	$8^x.\log 8$
12^x	$12^x.\log 12$
e^x	$e^x.\log_e e$ $= e^x \times 1 = e^x$
20^x	$20^x.\log 20$

$$\frac{d}{dx}(x^n) = n.x^{n-1}$$

$$\frac{d}{dx}(a^x) = a^x.\log a$$

$$\frac{d}{dx}(e^x) = e^x$$

⑦ $\frac{d}{dx} x^n = n.x^{n-1}$ $\Leftarrow$ variable$^{\text{constant}}$

$\frac{d}{dx} a^x = a^x . \log a$ $\Leftarrow$ constant$^{\text{variable}}$

$\frac{d}{dx} 21^x = 21^x . \log 21$

$\frac{d}{dx} x^{21} = 21 \times x^{20}$

$\frac{d}{dx} x^{50} = 50 . x^{49}$

$\frac{d}{dx} 50^x = 50^x . \log 50$

⑧

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\log x$	$1/x$
$\sqrt{x} = x^{1/2}$	$\frac{1}{2}(x)^{\frac{1}{2}-1}$ $= \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$
x	1
x^n	$n.x^{n-1}$
a^x	$a^x . \log a$
e^x	e^x
k = constant	0
10	0
25	0
36	0

⑨

$y = f(x)$	$\frac{dy}{dx} = f'(x)$	$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x	1	$8p+5$	0
x^2	$2x$	e^{15}	0
x^{35}	$35 . x^{34}$	x^{-88}	$-88 . x^{-89}$
19^x	$19^x . \log 19$	x^p	$p . (x)^{p-1}$
25^x	$25^x . \log 25$	p^x	$p^x . \log p$
e^x	e^x	x^k	$k . (x)^{k-1}$
38	0	k^x	$k^x . \log k$
$38p$	0	19^x	$19^x . \log 19$
45	0	x^{-19}	$-19 . x^{-20}$
x^{11}	$11 . x^{10}$	2^{35}	0
$\log x$	$1/x$	35^2	0
$\log 35$	0	35^x	$35^x . \log 35$
5^8	0	x^{35}	$35 . x^{34}$
$20^{1/2}$	0	$2p^2 + 19k$	0
$\sqrt{x}$	$1/2\sqrt{x}$	222^x	$222^x . \log 222$
$\sqrt{28}$	0	$x^{-3/7}$	$-3/7 \; x^{-10/7}$
$x^{3/2}$	$3/2 \; (x)^{1/2}$		
x^{-10}	$-10 . x^{-11}$	x^{-p-2}	$(-p-2) . x^{-p-3}$
x^{-35}	$-35 . x^{-36}$		
$x^{10/7}$	$10/7 \; (x)^{3/7}$		
$x^{-5/2}$	$-5/2 \; (x)^{-7/2}$		
39^{16}	0		
18^{-2}	0		
119^x	$119^x . \log 119$		
$\log e$	0		

(10) $\frac{d}{dx}(x^2) = 2x$ $\quad\quad$ $\frac{d}{dp} a^p = a^p \cdot \log a$

$\frac{d}{dx}(p^2) = 0$ $\quad\quad$ $\frac{d}{dp} e^p = e^p$

$\frac{d}{dp} p^2 = 2p$ $\quad\quad$ $\frac{d}{dx} e^p = 0$

$\frac{d}{dp}(x^2) = 0$ $\quad\quad$ $\frac{d}{dp} \log p = 1/p$

(11) (1) $\frac{d}{dx}(x^2 + 35^x) = \frac{d}{dx} x^2 + \frac{d}{dx} 35^x$

$= 2x + 35^x \cdot \log 35$

(2) $\frac{d}{dx}(\sqrt{x} - e^x) = \frac{d}{dx}\sqrt{x} - \frac{d}{dx} e^x$

$= \frac{1}{2\sqrt{x}} - e^x$

(3) $\frac{d}{dx}(x^{39} - 39^x) = \frac{d}{dx} x^{39} - \frac{d}{dx} 39^x$

$= 39 \cdot x^{38} - 39^x \cdot \log 39$

(4) $\frac{d}{dx}(\log x - x^{9/2}) = \frac{d}{dx}(\log x) - \frac{d}{dx} x^{9/2}$

$= \left[\frac{1}{x} - \frac{9}{2}(x)^{7/2}\right]$

(5) $\frac{d}{dx}\left(35^x + x^{35} - \sqrt{x} + x^{2/3}\right)$

$= \frac{d}{dx} 35^x + \frac{d}{dx} x^{35} - \frac{d}{dx}\sqrt{x} + \frac{d}{dx} x^{2/3}$

$= 35^x \cdot \log 35 + 35 \cdot x^{34} - \frac{1}{2\sqrt{x}} + \frac{2}{3}(x)^{-1/3}$

(12)

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$
$$= f'(x) + g'(x)$$

$$\frac{d}{dx}\left[f(x)-g(x)\right] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$
$$= f'(x) - g'(x)$$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

where u, v are functions of x

(13) $\frac{d}{dx}(x^5) =$

$$\frac{d}{dx}(u \times v)$$
$$= u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(x^3 \times x^2)$$
$$= x^3 \times \frac{d}{dx}x^2 + x^2 \cdot \frac{d}{dx}x^3$$
$$= x^3 \cdot 2x + x^2 \cdot 3x^2$$
$$= 2x^4 + 3x^4$$
$$= 5x^4$$

$$\frac{d}{dx}\left[f(x) \times g(x)\right]$$
$$= f(x) \times \frac{d}{dx}g(x) + g(x) \cdot \frac{d}{dx}f(x)$$
$$= f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

(14) $\frac{d}{dx}\left(5^{x} \times \sqrt{x}\right)$

$= 5^{x} \times \frac{d}{dx}\sqrt{x} + \sqrt{x} \cdot \frac{d}{dx} 5^{x}$

$= 5^{x} \cdot \frac{1}{2\sqrt{x}} + \sqrt{x} \cdot 5^{x} \cdot \log 5$

$= 5^{x}\left(\frac{1}{2\sqrt{x}} + \sqrt{x} \cdot \log 5\right)$

(15) $\frac{d}{dx}\left(x^{10}\right) =$

$\frac{d}{dx}\left(\frac{x^{15}}{x^{5}}\right)$

$= \left[\frac{x^{5} \times \frac{d}{dx} x^{15} - x^{15} \cdot \frac{d}{dx} x^{5}}{\left(x^{5}\right)^{2}}\right]$

$= \left(\frac{x^{5} \cdot 15 \cdot x^{14} - x^{15} \cdot 5 x^{4}}{x^{10}}\right)$

$= \left(\frac{15 x^{19} - 5 \cdot x^{19}}{x^{10}}\right) = \left(\frac{10 x^{19}}{x^{10}}\right)$

$= 10 (x)^{19-10} = 10 \cdot x^{9}$

i) $\frac{d}{dx}\left(\frac{u}{v}\right)$

$= \left[\frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^{2}}\right]$

ii) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$

$= \left[\frac{g(x) \times f'(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^{2}}\right]$

(16) when u, v are 2 diff functions of x then

$$\boxed{\begin{aligned} \frac{d}{dx}(u+v) &= \frac{du}{dx} + \frac{dv}{dx} \\ \frac{d}{dx}(u-v) &= \frac{du}{dx} - \frac{dv}{dx} \\ \frac{d}{dx}(u \times v) &= \left(u . \frac{dv}{dx} + v . \frac{du}{dx}\right) \\ \frac{d}{dx}\left(\frac{u}{v}\right) &= \left(\frac{v . \frac{du}{dx} - u . \frac{dv}{dx}}{v^2}\right) \end{aligned}}$$

(17) $\frac{d}{dx}(a^x \times e^x)$

$$= a^x \times \frac{d}{dx} e^x + e^x . \frac{d}{dx} a^x$$

$$= a^x . e^x + e^x . a^x . \log a$$

$$= a^x . e^x (1 + \log a)$$

(18) $\frac{d}{dx}\left(\sqrt{x} . \log x\right)$

$$= \sqrt{x} . \frac{d}{dx} \log x + \log x . \frac{d}{dx} \sqrt{x}$$

$$= \left(\sqrt{x} \times \frac{1}{x}\right) + \left(\log x \times \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{x}} + \frac{\log x}{2\sqrt{x}}$$

$$= \left(\frac{2 + \log x}{2\sqrt{x}}\right)$$

(19) $\frac{d}{dx}\left(\frac{a^x}{e^x}\right)$

$\Longrightarrow$

$$= \frac{e^x \times \frac{d}{dx} a^x - a^x \cdot \frac{d}{dx} e^x}{(e^x)^2}$$

$$= \left[\frac{e^x \cdot a^x \cdot \log a - a^x \cdot e^x}{(e^x)^2}\right] = \left[\frac{a^x \cdot e^x(\log a - 1)}{e^x \times e^x}\right]$$

$$= \left[\frac{a^x(\log a - 1)}{e^x}\right]$$

(20) $\frac{d}{dp}(a^p) = a^p \cdot \log a$

$\frac{d}{dx}(a^p) = 0$

$\frac{d}{dp}(\sqrt{p}) = 1/2\sqrt{p}$

$\frac{d}{dx}(\sqrt{p}) = 0$

$\frac{d}{dp}(p^n) = n \cdot p^{n-1}$

$\frac{d}{dx}(p^n) = 0$

$\frac{d}{dp}(\log p) = 1/p$

$\frac{d}{dx}(\log p) = 0$

(21)

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^2}$$

(22)

$$\frac{d}{dx}\left(10x^2\right)$$

$$= 10 \times \frac{d}{dx} x^2 + x^2 \times \frac{d}{dx}(10)$$

$$= 10 \times \frac{d}{dx} x^2 + \left(x^2 \times 0\right)$$

$$= 10 \times \frac{d}{dx} x^2$$

$$= 10 \times 2x = 20x$$

$$\frac{d}{dx}\left[k \times f(x)\right]$$

$$= k \times \frac{d}{dx} f(x)$$

(23)

$$\frac{d}{dx}\left(5 . \text{Log}x\right)$$

$$= 5 \times \frac{d}{dx} \text{Log}x + \text{Log}x \times \frac{d}{dx} 5$$

$$= 5 \times \frac{1}{x} + 0$$

$$= \frac{5}{x}$$

(24)

$$\frac{d}{dx} 18\sqrt{x}$$

$$= \left(18 \times \frac{1}{2\sqrt{x}}\right) = \frac{9}{\sqrt{x}} = 9(x)^{-1/2}$$

(25) $\frac{d}{dx}(8x^2+13x+1)$

$= \frac{d}{dx}8x^2 + \frac{d}{dx}13x + \frac{d}{dx}1$

$= (8\times 2x) + (13\times 1) + 0$

$= 16x + 13$

(26) $\frac{d}{dx}(10x^2 - 25x + 13)$

$= 20x - 25$

(27) If $y = \left(\frac{13x+1}{5x+3}\right)$ Find $\left(\frac{dy}{dx}\right)$

$\Longrightarrow$ $y = \left(\frac{13x+1}{5x+3}\right)$

Taking derivative on both sides with respect to x,

$$\frac{dy}{dx} = \frac{(5x+3)\cdot\frac{d}{dx}(13x+1) - (13x+1)\cdot\frac{d}{dx}(5x+3)}{(5x+3)^2}$$

$$\frac{dy}{dx} = \left[\frac{(5x+3)13 - (13x+1)5}{(5x+3)^2}\right]$$

$$\frac{dy}{dx} = \left[\frac{65\cancel{x} + 39 - 65\cancel{x} - 5}{(5x+3)^2}\right] = \left[\frac{34}{(5x+3)^2}\right]$$

(28) If $y = \left(\frac{5x-12}{12-13x}\right)$ Find $\frac{dy}{dx}$

$\Longrightarrow$

$$\frac{dy}{dx} = \frac{(12-13x)5 - (5x-12)(-13)}{(12-13x)^2}$$

$$= \frac{60 - \cancel{65x} + \cancel{65x} - 156}{(12-13x)^2} = \left[\frac{-96}{(12-13x)^2}\right]$$

(29) Find $\frac{dy}{dx}$ If

i) $y = 8x^2 - 13x + 22$

$dy/dx = 16x - 13$

ii) $y = 2\log x + 3e^x - 2x^2$

$$\frac{dy}{dx} = 2/x + 3e^x - 4x$$

iii) $y = \left(\frac{2x + 15x^2 - 13x^3 - 38x^4}{8}\right)$

$$\frac{dy}{dx} = \frac{1}{8}\left(2 + 30x - 39x^2 - 152x^3\right)$$

iv) $y = 20x^5 - 18x^4 - 22x^3 + 12x^2 - 2x + 13p - 19$

$$\frac{dy}{dx} = 100x^4 - 72x^3 - 66x^2 + 24x - 2$$

v) $y = (a^x \times \log x) + \log x$

$$\frac{dy}{dx} = \frac{d}{dx}(a^x . \log x) + \frac{d}{dx}(\log x)$$

$$= \left[a^x \times \frac{1}{x} + \log x . a^x . \log a\right] + \frac{1}{x}$$

$$= a^x\left(\frac{1}{x} + \log x . \log a\right) + \frac{1}{x}$$

(30) If $y = \left(\frac{13x^2 - 2x + 3}{2x + 19}\right)$ Find $\frac{dy}{dx}$

$\Longrightarrow$

$$\frac{dy}{dx} = \frac{(2x+19)(26x-2) - (13x^2 - 2x + 3)2}{(2x+19)^2}$$

$$= \left[\frac{52x^2 - \cancel{4x} + 494x - 38 - 26x^2 + \cancel{4x} - 6}{(2x+19)^2}\right]$$

$$= \left[\frac{26x^2 + 494x - 44}{(2x+19)^2}\right]$$

(31) If $f(x) = \left(\frac{2x+13}{a^x}\right)$ Find $f'(x)$

$$\Longrightarrow \quad f'(x) = \frac{a^x \times 2 - (2x+13) \cdot a^x \cdot \log a}{(a^x)^2}$$

$$= \frac{a^x\left[2 - \log a \times (2x+13)\right]}{a^x \times a^x}$$

$$= \left[\frac{2 - \log a \times (2x+13)}{a^x}\right]$$

(32) $\frac{d}{dp}\left(25p^5 - 18p^3 + 28p^2 + 39p^4 - 18p - 23\right)$

$= 125p^4 - 54p^2 + 56p + 156p^3 - 18$

(33) If $3x + 5y = 23$ Find $\frac{dy}{dx}$

$\Longrightarrow$ $3x + 5y = 23$

Taking derivative on both sides w.r.to x

$\frac{d}{dx}(3x + 5y) = \frac{d}{dx}(23)$

$\frac{d}{dx} 3x + \frac{d}{dx} 5y = 0$

$3 \times \frac{d}{dx} x + 5 \times \frac{dy}{dx} = 0$

$3 + 5 . \frac{dy}{dx} = 0$

$5 \frac{dy}{dx} = -3$

$\frac{dy}{dx} = -3/5$

OR

$3x + 5y = 23$

$5y = 23 - 3x$

$y = \frac{23}{5} - \frac{3}{5}x$

Taking derivative on both sides w.r.to x

$\frac{dy}{dx} = 0 - \frac{3}{5} \times 1$

$= -3/5$

Here $\left(\frac{dy}{dx}\right)$ represents slope of the line.

(34) If $y = 5 \operatorname{Log} x - 8e^{x} + 15 \times 5^{x} - 8x^{2} + 35$

Find $\frac{dy}{dx}$

$\Longrightarrow$ $\frac{dy}{dx} = \frac{5}{x} - 8e^{x} + 15 \times 5^{x} \times \operatorname{Log} 5 - 16x + 0$

(35) If $y = (13x^3 - 2x^2 + 13x + 2)$ Find $\left(\frac{dy}{dx}\right)_{x=2}$

$\Longrightarrow$ $\frac{dy}{dx} = 39x^2 - 4x + 13$

$\left(\frac{dy}{dx}\right)_{x=2} = 39(2)^2 - 4(2) + 13 = 161$

(36) If $y = (20x^3 - 55x^2 - 22x + 22)$

Find $\left(\frac{dy}{dx}\right)_{x=-3}$

$\Longrightarrow$

$$\frac{dy}{dx} = 60x^2 - 110x - 22$$

$$\left(\frac{dy}{dx}\right)_{x=-3} = 60(-3)^2 - 110(-3) - 22$$

$$= (60 \times 9) + 330 - 22$$

$$= 848$$

(37) If $y = (5x + 13)^2$

$$\frac{dy}{dx} = 2 \times (5x+13)^1 \times \frac{d}{dx}(5x+13)$$

$$= 2 \times (5x+13) \times 5$$

$$= 10\ (5x+13)$$

$$= 50x + 130$$

$$\frac{d}{dx}(x)^n = n(x)^{n-1} \times \frac{d}{dx}x$$

$$= n \times (x)^{n-1}$$

$$\frac{d}{dx}[f(x)]^n$$

$$= n\,[f(x)]^{n-1} \times f'(x)$$

cross-check : $y = (5x+13)^2 = 25x^2 + 130x + 169$

$$\therefore \frac{dy}{dx} = 50x + 130$$

(38) $y = (5x^3 - 2x^2 + 18x + 3)^8$ Find $\frac{dy}{dx}$

$\Longrightarrow$ $\frac{dy}{dx} = 8 \times (5x^3 - 2x^2 + 18x + 3)^7 \times (15x^2 - 4x + 18)$

(39) $y = (8x+3)^4$ Find $\frac{dy}{dx}$

$\Longrightarrow$ $\frac{dy}{dx} = 4 \times (8x+3)^3 \times 8 = 32 \times (8x+3)^3$

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$\frac{d}{dx}(x)^n = n(x)^{n-1} \times \frac{d}{dx}(x)$ $= n(x)^{n-1}$	$\frac{d}{dx}[f(x)]^n$ $= n \times [f(x)]^{n-1} \times f'(x)$
$\frac{d}{dx} a^x = a^x \times \log a \times \frac{d}{dx} x$ $= a^x . \log a$	$\frac{d}{dx}(a)^{f(x)}$ $= a^{f(x)} . \log a . f'(x)$
$\frac{d}{dx} e^x = e^x \times \frac{d}{dx} x$ $= e^x$	$\frac{d}{dx} e^{f(x)}$ $= e^{f(x)} . f'(x)$
$\frac{d}{dx} \log x = \frac{1}{x} \times \frac{d}{dx}(x)$ $= \frac{1}{x}$	$\frac{d}{dx} \log [f(x)]$ $= \frac{1}{f(x)} \times f'(x)$
$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \times \frac{d}{dx} x$ $= \frac{1}{2\sqrt{x}}$	$\frac{d}{dx} \sqrt{f(x)}$ $= \frac{1}{2\sqrt{f(x)}} \times f'(x)$

41 i) $\frac{d}{dx} 5^{(8x+10)}$

$= 5^{8x+10} . \log 5 . 8$

ii) $\frac{d}{dx}(2x+3)^n$

$= n(2x+3)^{n-1} \times 2$

iii) $\frac{d}{dx}(5)^{\sqrt{x}}$

$= 5^{\sqrt{x}} \cdot \log 5 \cdot \frac{1}{2\sqrt{x}}$

iv) $\frac{d}{dx} \log(2x^2+8x-19)$

$= \frac{1}{(2x^2+8x-19)} \times (4x+8) = \left(\frac{4x+8}{2x^2+8x-19}\right)$

v) $\frac{d}{dx} \sqrt{\log x}$

$= \frac{1}{2\sqrt{\log x}} \times \frac{1}{x} = \frac{1}{2x\sqrt{\log x}}$

vi) $\frac{d}{dx} e^{\log x + a^x - e^x - 35x}$

$= e^{\log x + a^x - e^x - 35x} \cdot \left(\frac{1}{x} + a^x \cdot \log a - e^x - 35\right)$

vii) $\frac{d}{dx} \log\sqrt{10x+23}$

$= \frac{1}{\sqrt{10x+23}} \times \frac{d}{dx}\left(\sqrt{10x+23}\right)$

$= \frac{1}{\sqrt{10x+23}} \times \frac{1}{2\sqrt{10x+23}} \times \frac{d}{dx}(10x+23)$

$= \frac{1}{2(10x+23)} \times 10 = \left(\frac{5}{10x+23}\right)$

(42) Find $\frac{dy}{dx}$

$y = \log(5x+28)$

$\Rightarrow$ By using Rule

$\frac{d}{dx}\log[f(x)] = \frac{1}{f(x)} \times f'(x)$

$y = \log(5x+28)$

$\therefore \frac{dy}{dx} = \frac{1}{5x+28} \times \frac{d}{dx}(5x+28)$

$\frac{dy}{dx} = \left(\frac{5}{5x+28}\right)$

OR

$y = \log(5x+28)$

$t = 5x+28 \quad \therefore y = \log t$

$\frac{dt}{dx} = 5, \quad \frac{dy}{dt} = \frac{1}{t}$

By using chain rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{1}{t} \times 5$

$\frac{dy}{dx} = \frac{5}{t} = \frac{5}{(5x+28)}$

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(43) $y = (13x-3)^{11}$ Find $\frac{dy}{dx}$

$\Rightarrow$

$\frac{dy}{dx} = 11 \times (13x-3)^{10} \times 13$

$= 143 \times (13x-3)^{10}$

OR suppose

$t = 13x-3 \quad \therefore y = t^{11}$

$\frac{dt}{dx} = 13, \quad \frac{dy}{dt} = 11 \cdot t^{10}$

As per chain rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= 11 \times (t)^{10} \times 13$

$= 143\,(t)^{10}$

$= 143\,(13x-3)^{10}$

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$$\frac{d}{dx}\left[f(x)\right]^n = n \times \left[f(x)\right]^{n-1} \times f'(x)$$

$$\frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot \log a \cdot f'(x)$$

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$$

$$\frac{d}{dx} \log f(x) = \frac{1}{f(x)} \times f'(x)$$

$$\frac{d}{dx} \sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}} \times f'(x)$$

45 As per chain Rule

Derivative of y with respect to x = [Derivative of y with respect to t × Derivative of t with respect to x]

$y = \sqrt{5x+3}$

Find $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{5x+3}} \times 5$$

$$= \frac{5}{2\sqrt{5x+3}}$$

$\Downarrow$

By using

$$\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}} \times f'(x)$$

OR By chain rule

$t = 5x+3 \quad y = \sqrt{t}$

$$\frac{dt}{dx} = 5 \ \& \ \frac{dy}{dt} = \frac{1}{2\sqrt{t}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{1}{2\sqrt{t}} \times 5$$

$$= \frac{5}{2\sqrt{5x+3}}$$

(46) If $y = \left(\frac{1+t^2}{2}\right)$, $x = t^3 - t$

Find $\frac{dy}{dx}$

$\Longrightarrow$

$y = \frac{1+t^2}{2}$

$\frac{dy}{dt} = \frac{1}{2}(0+2t)$

$= t$

$x = t^3 - t$

$\frac{dx}{dt} = 3t^2 - 1$

As per chain rule

$$\frac{dy}{dx} = \left(\frac{dy/dt}{dx/dt}\right) = \left(\frac{t}{3t^2-1}\right)$$

(47) If $y = 5t^2 - 2t + 1$, $x = 8t^2 - 13t + 13$

Find $\left(\frac{dy}{dx}\right)_{t=10}$

$\Longrightarrow$

$y = 5t^2 - 2t + 1$

$\frac{dy}{dt} = 10t - 2$

$x = 8t^2 - 13t + 13$

$\frac{dx}{dt} = 16t - 13$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t-2}{16t-13}$$

$$\left(\frac{dy}{dx}\right)_{t=10} = \frac{10 \times 10 - 2}{16 \times 10 - 13} = \left(\frac{98}{147}\right) = 2/3$$

(48) If $y = \left(\frac{1+t^2}{1-t^2}\right)$ $x = \left(\frac{2at}{1-t^2}\right)$ Find $\frac{dy}{dx}$

$\Longrightarrow$

$y = \frac{1+t^2}{1-t^2}$ $\quad \therefore \frac{dy}{dt} = \frac{(1-t^2)(2t) - (1+t^2)(-2t)}{(1-t^2)^2}$

$$\frac{dy}{dt} = \frac{2t - \cancel{2t^3} + 2t + \cancel{2t^3}}{(1-t^2)^2}$$

$$\frac{dy}{dt} = \frac{4t}{(1-t^2)^2}$$

$x = \left(\frac{2at}{1-t^2}\right)$ $\quad \therefore \frac{dx}{dt} = \frac{(1-t^2)2a - 2at \times (-2t)}{(1-t^2)^2}$

$$\frac{dx}{dt} = \frac{2a - 2at^2 + 4at^2}{(1-t^2)^2}$$

$$\frac{dx}{dt} = \frac{2a + 2at^2}{(1-t^2)^2} = \frac{2a(1+t^2)}{(1-t^2)^2}$$

As per chain rule $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t}{2a(1+t^2)}$

$$= \frac{2t}{a(1+t^2)}$$

(49) $y = 3t^3 - 2t + 9$, $x = \log t$

Find $\left(\frac{dy}{dx}\right)$

$\Longrightarrow$ $\frac{dy}{dt} = 9t^2 - 2$ $\quad \therefore \frac{dy}{dx} = \frac{9t^2 - 2}{1/t}$

$\frac{dx}{dt} = \frac{1}{t}$ $\quad = 9t^3 - 2t$

(50) Find $f'(x)$ If

i) $f(x) = 3x^2 + 5x - 2$

ii) $f(x) = a^x + x^a + a^a$

iii) $f(x) = \frac{1}{3}x^3 + 5x^2 + 6x - 2\log x + 3$

iv) $f(x) = e^x . \log x$

v) $f(x) = 2^x - x^5$

vi) $f(x) = \left(\frac{x^2}{e^x}\right)$

vii) $f(x) = e^x / \log x$

viii) $f(x) = 2^x . \log x$

ix) $f(x) = \left(\frac{2x}{3x^3+7}\right)$

$\Longrightarrow$

i) $f'(x) = 6x + 5$

ii) $f'(x) = a^x . \log a + a . x^{a-1} + 0$

$= a^x . \log a + a . x^{a-1}$

iii) $f'(x) = \frac{1}{3} \times 3x^2 + 5 \times 2x + 6 \times 1 - 2 \times \frac{1}{x} + 0$

$= x^2 + 10x + 6 - \frac{2}{x}$

iv) $f'(x) = e^x \times \frac{1}{x} + \log x . e^x$

$= e^x \left(\frac{1}{x} + \log x\right)$

v) $f'(x) = 2^x . \log 2 - 5x^4$

vi) $f'(x) = \frac{e^x \times 2x - x^2 . e^x}{(e^x)^2} = \left(\frac{2x - x^2}{e^x}\right)$

vii) $f'(x) = \frac{\log x . e^x - e^x . \frac{1}{x}}{(\log x)^2} = \frac{e^x \left(\log x - \frac{1}{x}\right)}{(\log x)^2}$

viii) $f'(x) = 2^x \times \frac{1}{x} + \log x . 2^x . \log 2$

$= 2^x \left(\frac{1}{x} + \log x . \log 2\right)$

ix) $f'(x) = \dfrac{(3x^3+7)2 - 2x \times (9x^2)}{(3x^3+7)^2}$

$= \dfrac{6x^3 + 14 - 18x^3}{(3x^3+7)^2}$

$= \dfrac{14 - 12x^3}{(3x^3+7)^2}$

(51) If $y = x^x$. Find $\dfrac{dy}{dx}$

$\Longrightarrow$ $y = x^x$

Taking log on both sides

$\log y = \log x^x$

$\log y = x \cdot \log x$

Taking derivative on both sides w.r.to x

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = x \times \dfrac{1}{x} + \log x \times 1$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = 1 + \log x$

$\dfrac{dy}{dx} = y(1+\log x)$

$= x^x(1+\log x)$

(52) $y = 5^x + x^5 + 5^5 + x^x$. Find $\dfrac{dy}{dx}$

$\Longrightarrow$ $\dfrac{dy}{dx} = 5^x \cdot \log 5 + 5 \cdot x^4 + 0 + x^x(1+\log x)$

(53) $y = x^{\log x}$. Find $\frac{dy}{dx}$

$\Longrightarrow$

$$y = x^{\log x}$$

$$\log y = \log x^{\log x}$$

$$\log y = \log x . \log x$$

$$\log y = (\log x)^2$$

$$\frac{1}{y} . \frac{dy}{dx} = 2 \times (\log x)^1 \times \frac{1}{x}$$

$$\frac{1}{y} . \frac{dy}{dx} = \frac{2 \log x}{x}$$

$$\frac{dy}{dx} = x^{\log x} \times \frac{2 \log x}{x^1}$$

$$= x^{\log x - 1} . 2 \log x$$

(54) $y = (8x+3)^x$ Find $\frac{dy}{dx}$

$\Longrightarrow$

$$y = (8x+3)^x$$

$$\log y = \log (8x+3)^x$$

$$\log y = x . \log (8x+3)$$

$$\frac{1}{y} . \frac{dy}{dx} = x \times \frac{1}{8x+3} \times 8 + \log(8x+3) \times 1$$

$$\frac{1}{y} . \frac{dy}{dx} = \frac{8x}{8x+3} + \log(8x+3)$$

$$\frac{dy}{dx} = (8x+3)^x \left[\frac{8x}{8x+3} + \log(8x+3) \right]$$

55 If $y = 2at - t^2$, $x = 4bt - t^3$ Find $\left(\frac{dy}{dx}\right)_{t=3}$ } Derivatives of parametric Equations

$\Longrightarrow$ $y = 2at - t^2$ $\therefore \frac{dy}{dt} = 2a - 2t$

$x = 4bt - t^3$ $\therefore \frac{dx}{dt} = 4b - 3t^2$

As per chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left(\frac{2a-2t}{4b-3t^2}\right)$$

$$\left(\frac{dy}{dx}\right)_{t=3} = \frac{2a - 2\times3}{4b - 3\,(3)^2} = \frac{2a-6}{4b-27}$$

$$= \left[\frac{2\,(a-3)}{4b-27}\right]$$

56 If $3xy - y^2 = 5y$. Find $\frac{dy}{dx}$ } Derivaties of implicit functions

$\Longrightarrow$ $3xy - y^2 = 5y$

Taking deri. on both sides w.r.to x

$$\frac{d}{dx}3xy - \frac{d}{dx}y^2 = \frac{d}{dx}5y$$

$$3\left(x \times \frac{dy}{dx} + y \times 1\right) - 2y.\frac{dy}{dx} = 5\frac{dy}{dx}$$

$$3x \cdot \frac{dy}{dx} + 3y - 2y \cdot \frac{dy}{dx} - 5 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3x - 2y - 5) = -3y$$

$$\frac{dy}{dx} = \left(\frac{-3y}{3x - 2y - 5}\right) = \left(\frac{3y}{5 + 2y - 3x}\right)$$

57 If $3x^2y^2 - 2y = 8$ Find $\frac{dy}{dx}$

$\Longrightarrow$

$$3x^2y^2 - 2y = 8$$

$$3 \cdot \frac{d}{dx}(x^2y^2) - 2 \times \frac{dy}{dx} = 0$$

$$3\left(x^2 \times 2y \cdot \frac{dy}{dx} + y^2 \times 2x\right) - 2 \cdot \frac{dy}{dx} = 0$$

$$6x^2y \cdot \frac{dy}{dx} + 6xy^2 - \frac{2dy}{dx} = 0$$

$$\frac{dy}{dx}(6x^2y - 2) = -6xy^2$$

$$\frac{dy}{dx} = \frac{-6xy^2}{6x^2y - 2} = \frac{-3xy^2}{3x^2y - 1}$$

$$\frac{dy}{dx} = \left(\frac{3xy^2}{1 - 3x^2y}\right)$$

58 If $xy = 10$. Find $\frac{dy}{dx}$

$\Longrightarrow$

$$xy = 10$$

$$y = \frac{10}{x}$$

$$y = 10 \cdot x^{-1}$$

$$\frac{dy}{dx} = 10 \times \frac{d}{dx} x^{-1}$$

$$= 10 \times -1 \times x^{-1-1} = 10 \times \left(-1 \times x^{-2}\right)$$

$$= \left(\frac{-10}{x^2}\right)$$

OR $\quad xy = 10$

$$x \cdot \frac{dy}{dx} + y = 0$$

$$x \cdot \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x} = -\left(\frac{10/x}{x}\right) = \left(\frac{-10}{x^2}\right)$$

59 $\quad y = \sqrt{\left(\frac{1+x}{1-x}\right)}$ Find $\frac{dy}{dx}$

$$\Longrightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \times \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2} \times \sqrt{\frac{1-x}{1+x}} \times \frac{(1-x) \times 1 - (1+x)(-1)}{(1-x)^2}$$

$$= \frac{1}{2} \times \frac{(1-x)^{1/2}}{(1+x)^{1/2}} \times \frac{1 - \cancel{x} + 1 + \cancel{x}}{(1-x)^2}$$

$$= \frac{1}{\cancel{2}} \times \frac{(1-x)^{\frac{1}{2}-2}}{(1+x)^{1/2}} \times \cancel{2} = \frac{(1-x)^{-3/2}}{(1+x)^{1/2}}$$

$$= \sqrt{\frac{(1-x)^{-3}}{(1+x)}}$$

(60) If $y = f(x) = 18x^4 - 20x^3 + 25x^2 - 9x + 13$

Find $f''(x) = \frac{d^2y}{dx^2}$: Higher order derivative

$\Longrightarrow$

$$y = f(x) = 18x^4 - 20x^3 + 25x^2 - 9x + 13$$

$$\therefore \frac{dy}{dx} = f'(x) = 72x^3 - 60x^2 + 50x - 9$$

$$\frac{d^2y}{dx^2} = f''(x) = 216x^2 - 120x + 50$$ } second order derivative

$$\frac{d^3y}{dx^3} = f'''(x) = 432x - 120$$ } Third order derivative

(61) If $y = 8x^5 - 20x^3 + 12x^4 - 19x + 28$

Find $\left(\frac{d^2y}{dx^2}\right)$, $\left(\frac{d^2y}{dx^2}\right)_{x=2}$

$\Longrightarrow$

$$\frac{dy}{dx} = 40x^4 - 60x^2 + 48x^3 - 19$$

$$\frac{d^2y}{dx^2} = 160x^3 - 120x + 144x^2$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=2} = 160(2)^3 - 120(2) + 144(2)^2$$

$$= 1616$$

(62) Find the gradient of curve $y = 3x^2 - 5x + 4$ at point $(1,2)$

$\Longrightarrow$

$$\frac{dy}{dx} = 6x - 5$$

$$\left(\frac{dy}{dx}\right)_{x=1, y=2} = 6(1) - 5 = 1$$

(63) If $f(x) = \text{Log}\left[\sqrt{x^2+a^2} + \sqrt{x}\right]$ Find $f'(x)$

$\Longrightarrow$

By using rule

$y = \text{Log } f(x)$

then $\dfrac{dy}{dx} = \dfrac{1}{f(x)} \times f'(x)$

$$f'(x) = \frac{1}{\sqrt{x^2+a^2} + \sqrt{x}} \cdot \frac{d}{dx}\left(\sqrt{x^2+a^2} + \sqrt{x}\right)$$

$$= \frac{1}{\sqrt{x^2+a^2} + \sqrt{x}} \times \left(\frac{1}{\cancel{2}\sqrt{x^2+a^2}} \times \cancel{2}x + \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{x^2+a^2} + \sqrt{x}} \times \frac{2x\sqrt{x} + \sqrt{x^2+a^2}}{2\sqrt{x}\sqrt{x^2+a^2}}$$

(64) $y = at^3$, $x = 2bt$ Find $\left(\dfrac{dy}{dx}\right)$, $\left(\dfrac{dy}{dx}\right)_{t=8}$

$\Longrightarrow$

$$\frac{dy}{dt} = a \times 3t^2 = 3at^2$$

$$\frac{dx}{dt} = 2b$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left(\frac{3at^2}{2b}\right)$$

$$\left(\frac{dy}{dx}\right)_{t=8} = \frac{192a}{2b} = \left(\frac{96a}{b}\right)$$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\log(5x+3)$	$\frac{1}{5x+3} \times 5 = \left(\frac{5}{5x+3}\right)$
$\log(2x^2+3x+1)$	$\frac{1}{2x^2+3x+1} \times 4x+3 = \left(\frac{4x+3}{2x^2+3x+1}\right)$
$\sqrt{2x^2+3}$	$= \frac{1}{2\sqrt{2x^2+3}} \times 4x = \frac{2x}{\sqrt{2x^2+3}}$
$\sqrt{\frac{1+x}{1-x}}$	$\frac{1}{2\sqrt{\frac{1+x}{1-x}}} \times \frac{(1-x)1+(1+x)}{(1-x)^2}$ $= \frac{1}{\cancel{2}} \times \sqrt{\frac{1-x}{1+x}} \times \frac{\cancel{2}}{(1-x)^2}$ $= \frac{(1-x)^{1/2}}{(1+x)^{1/2}} \times \frac{1}{(1-x)^2} = \frac{(1-x)^{-3/2}}{(1+x)^{1/2}}$
a^{5x+33}	$a^{5x+33} \cdot \log a \cdot 5$
e^{ax^2+bx+c}	$e^{ax^2+bx+c} \cdot (2ax+b)$
$(5x^2+19x+3)^{10}$	$10 \times (5x^2+19x+3)^9 \times (10x+19)$
x^x	$x^x(1+\log x)$
$\sqrt{a^{\log x}}$	$\frac{1}{2\sqrt{a^{\log x}}} \times a^{\log x} \times \log a \times \frac{1}{x}$

66) If $f(x) = x^k$ and $f'(1) = 0$ then $k = ?$

$\Longrightarrow$

$$f(x) = x^k$$
$$f'(x) = k.(x)^{k-1}$$
$$f'(1) = k(1)^{k-1} = k = 0$$

67) If $f(x) = 5x^2 + 8x + 19$

Find $f'(3)$, $f'(-10)$, $f'(k+1)$, $f'(-33)$

$\Longrightarrow$

$$f(x) = 5x^2 + 8x + 19$$
$$f'(x) = 10x + 8$$

i) $f'(3) = 10(3) + 8 = 38$

ii) $f'(-10) = 10(-10) + 8 = -92$

iii) $f'(k+1) = 10(k+1) + 8 = 10k + 18$

iv) $f'(-33) = 10(-33) + 8 = -322$

68) $y = x^{x^{x^{x^{x^{x^{\cdots}}}}}}$ ∞ terms   Find $\left(\frac{dy}{dx}\right)$

$\Longrightarrow$

$$y = x^{x^{x^{x^{x^{x^{\cdots}}}}}} \text{ ∞ terms}$$
$$y = (x)^y$$
$$\operatorname{Log} y = y . \operatorname{Log} x$$
$$\frac{1}{y} . \frac{dy}{dx} = y \times \frac{1}{x} + \operatorname{Log} x \times \frac{dy}{dx}$$
$$\frac{1}{y} . \frac{dy}{dx} - \operatorname{Log} x . \frac{dy}{dx} = \frac{y}{x}$$

$$\therefore \quad \frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1 - y.\log x}{y}\right) = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} \times \frac{y}{1 - y\log x} = \frac{y^2}{x(1 - y.\log x)}$$

(69) If $x^2y^2 + 3xy + y = 0$ Find $\left(\frac{dy}{dx}\right)$

$$\Longrightarrow \quad x^2y^2 + 3xy + y = 0$$

$$\frac{d}{dx}x^2y^2 + 3.\frac{d}{dx}xy + \frac{dy}{dx} = 0$$

$$x^2 \times 2y.\frac{dy}{dx} + y^2 \times 2x + 3\left(x.\frac{dy}{dx} + y\right) + \frac{dy}{dx} = 0$$

$$2x^2y.\frac{dy}{dx} + 2xy^2 + 3x.\frac{dy}{dx} + 3y + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2x^2y + 3x + 1\right) = -2xy^2 - 3y$$

$$\frac{dy}{dx} = \frac{-(2xy^2 + 3y)}{2x^2y + 3x + 1}$$

(70) If $f(x) = \left(\frac{x^2+1}{x^2-1}\right)$ Find $f'(10)$

$$\Longrightarrow \quad f'(x) = \frac{(x^2-1)2x - (x^2+1)2x}{(x^2-1)^2}$$

$$= \frac{2\cancel{x^3} - 2x - 2\cancel{x^3} - 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$f'(10) = \frac{-4 \times 10}{(10^2-1)^2} = \left(\frac{-40}{9801}\right)$$

(71) If $y = a.e^{mx} + b.e^{-mx}$ Find $\left(\frac{dy}{dx}\right)$ & $\left(\frac{d^2y}{dx^2}\right)$

$$\Longrightarrow \frac{dy}{dx} = a.e^{mx}.m + b.e^{-mx}.-m$$

$$= a.m.e^{mx} - bm.e^{-mx}$$

$$\frac{d^2y}{dx^2} = am.e^{mx}.m - bm.e^{-mx}.(-m)$$

$$= am^2.e^{mx} + bm^2.e^{-mx}$$

$$= m^2\left(a.e^{mx} + b.e^{-mx}\right)$$

$$= m^2y$$

(72) If $y = 8x^4 - 9x^3 + 33x^2 + 8x + 101 = f(x)$

Find $f''(-3)$

$$\Longrightarrow f(x) = 8x^4 - 9x^3 + 33x^2 + 8x + 101$$

$$f'(x) = 32x^3 - 27x^2 + 66x + 8$$

$$f''(x) = 96x^2 - 54x + 66$$

$$f''(-3) = 96(-3)^2 - 54(-3) + 66$$

$$= 1092$$

(73) If $y = \sqrt{x + \sqrt{x}}$ Find $\frac{dy}{dx}$

$$\Longrightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}} \times \frac{d}{dx}\left(x+\sqrt{x}\right)$$

$$\frac{dy}{dx} = \frac{1 + \dfrac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}} = \frac{\dfrac{2\sqrt{x}+1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x}+1}{4\sqrt{x}\cdot\sqrt{x+\sqrt{x}}} = \frac{2\sqrt{x}+1}{4\sqrt{x(x+\sqrt{x})}}$$

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$$\begin{tikzcd} f(x) \arrow[r, bend left, "\text{Derivative}"] & f'(x) \arrow[l, bend left, "\text{Integration}"] \end{tikzcd}$$

Integration is Anti_Derivative

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$$\frac{d}{dx}(x) = 1 \quad , \quad \int 1 . dx = x + c$$

$$\frac{d}{dx}(x+8) = 1 \quad , \quad \int 1 . dx = x + c$$

$$\frac{d}{dx} x^2 = 2x \quad , \quad \int x^n . dx = \left(\frac{x^{n+1}}{n+1}\right) + c$$

$$\int 2x . dx = 2 \times \int x^1 . dx$$

$$= \cancel{2} \times \frac{x^{1+1}}{(1+\cancel{1})} + c$$

$$= x^2 + c$$

(76) $$\boxed{\int x^n \cdot dx = \frac{x^{n+1}}{(n+1)} + c}$$

$$\int 3x^2 \cdot dx = 3 \times \int x^2 \cdot dx$$

$$= 3 \times \frac{x^3}{3} + c$$

$$= x^3 + c$$

(77) $$\int x^8 \, dx = \left(\frac{x^9}{9}\right) + c$$

Let's take $\frac{d}{dx}\left(\frac{x^9}{9} + c\right)$

$$= \frac{d}{dx}\left(\frac{x^9}{9}\right) + \frac{d}{dx}(c)$$

$$= \frac{1}{9} \times \frac{d}{dx} x^9 + 0$$

$$= \frac{1}{9} \times 9x^8 + 0$$

$$= x^8$$

(78) $$\int 5x^4 \cdot dx = 5 \times \frac{x^5}{5} + c$$

$$= x^5 + c$$

$$\int 10x^9 \cdot dx = 10 \times \frac{x^{10}}{10} + c$$

$$= x^{10} + c$$

$$\int 20 \cdot x^{19} \cdot dx = 20 \times \frac{x^{20}}{20} + c$$

$$= x^{20} + c$$

(79)

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt{x} \cdot dx = \frac{x^{3/2}}{3/2} + c$$

$$\int 1 \cdot dx = x + c \quad , \quad \int 1 \cdot dt = t + c$$

$$\int a^x \cdot dx = \frac{a^x}{\log a} + c$$

$$\int e^x \cdot dx = e^x + c$$

$$\int \frac{1}{x} \cdot dx = \log x + c$$

$$\int k \cdot dx = kx + c$$

$$\int k \cdot f(x) \cdot dx = k \times \int f(x) \cdot dx$$

(80)

$$\int (u + v) \cdot dx = \int u \cdot dx + \int v \cdot dx$$

$$\int (u - v) \cdot dx = \int u \cdot dx - \int v \cdot dx$$

u, v are functions of x

$$\int (u \times v)\, dx = u \times \int v \cdot dx - \int \left[\frac{du}{dx} \times \int v \cdot dx \right] \cdot dx$$

------ Integration by parts

(81) ① $\int 5^x \cdot dx = \frac{5^x}{\log 5} + c$

② $\int e^x \cdot dx = \frac{e^x}{\log_e e} + c = e^x + c$

③ $\int (x^9 + x^5 + 5x^3 + 2x + 19) \cdot dx$

$= \int x^9 \cdot dx + \int x^5 \cdot dx + 5 \int x^3 \cdot dx + 2 \int x \cdot dx + 19 \int 1 \cdot dx$

$= \frac{x^{10}}{10} + \frac{x^6}{6} + \frac{5x^4}{4} + \frac{2x^2}{2} + 19x + c$

(82) $\int (5x^3 - 13x^2 + 2x + 8) \cdot dx$

$= \frac{5x^4}{4} - \frac{13x^3}{3} + \frac{2x^2}{2} + 8x + c$

(83) $\int (20x^4 - 18x^5 + 13x^3 - 22x^2 + 100x + 35) \cdot dx$

$= \frac{20x^5}{5} - \frac{18x^6}{6} + \frac{13x^4}{4} - \frac{22x^3}{3} + \frac{100x^2}{2} + 35x + c$

$= 4x^5 - 3x^6 + \frac{13}{4}x^4 - \frac{22}{3}x^3 + 50x^2 + 35x + c$

(84) $$\int \frac{1}{ax+b}\,.dx = \frac{\log(ax+b)}{\frac{d}{dx}(ax+b)} + c$$

$$= \frac{\log(ax+b)}{a} + c$$

<u>cross-check</u>

$$\frac{d}{dx}\left[\frac{\log(ax+b)}{a}\right] = \frac{1}{\cancel{a}} \times \frac{1}{ax+b} \times \cancel{a}$$

$$= \left(\frac{1}{ax+b}\right)$$

$$\boxed{\int \frac{1}{px+q}\,.dx = \left(\frac{\log(px+q)}{p}\right) + c}$$

(85) ① $$\int \left(\frac{10}{8x+33}\right) dx = 10 \times \frac{\log(8x+33)}{8} + c$$

$$= \frac{5}{4}\log(8x+33) + c$$

② $$\int \frac{22}{22x+19}\,.dx = 22 \times \frac{\log(22x+19)}{22} + c$$

$$= \log(22x+19) + c$$

③ $$\int \frac{2p}{8px+33}\,.dx = 2p \times \frac{\log(8px+33)}{8p} + c$$

$$= \frac{\log(8px+33)}{4} + c$$

(86) $\int\left(5x^{10}+22^{x}-e^{x}+\frac{10}{5x+3}\right).dx$

$= \int 5.x^{10}.dx + \int 22^{x}.dx - \int e^{x}.dx + \int \frac{10}{5x+3}.dx$

$= \frac{5x^{11}}{11} + \frac{22^{x}}{\log 22} - e^{x} + \frac{10\log(5x+3)}{5} + c$

$= \frac{5}{11}x^{11} + \frac{22^{x}}{\log 22} - e^{x} + 2\log(5x+3) + c$

(87) $\int e^{5x}.dx$

$\Longrightarrow \int e^{5x}.dx$

$= \left(\frac{e^{5x}}{5}\right) + c$

(88) $\int e^{5x+3}.dx$

$\Longrightarrow \int e^{5x+3}.dx$

$= \left(\frac{e^{5x+3}}{5}\right) + c$

(89) $\int a^{10x+16}.dx$

$\Longrightarrow \int a^{10x+16}.dx$

$= \frac{a^{10x+16}}{\log a \times 10} + c$

cross-check

$\frac{d}{dx}\left(\frac{a^{10x+16}}{10.\log a} + c\right)$

$= \frac{1}{\cancel{10}\,\cancel{\log a}} \times a^{10x+16}.\cancel{\log a}.\cancel{10} + 0$

$= a^{10x+16}$

(90) $\int \left(8x^{-3} + 2x^2 + 19/x^5 + \sqrt{x} + e^x + \frac{1}{x}\right) . dx$

$= 8\int x^{-3}. dx + 2\int x^2 . dx + 19 \int x^{-5}. dx + \int x^{1/2}. dx + \int e^x. dx + \int \frac{1}{x}. dx$

$= 8 \times \frac{x^{-2}}{-2} + \frac{2x^3}{3} + \frac{19x^{-4}}{-4} + \frac{x^{3/2}}{3/2} + e^x + \text{Log } x + c$

$= \frac{-4}{x^2} + \frac{2}{3}x^3 - \frac{19}{4x^4} + \frac{2x^{3/2}}{3} + e^x + \text{Log } x + c$

(91) $\int \sqrt{x} \left(x^3 - 3x^2 + 2\right) . dx$

$\Rightarrow \quad = \int x^{1/2} \left(x^3 - 3x^2 + 2\right) . dx$

$= \int \left(x^{7/2} - 3x^{5/2} + 2x^{1/2}\right) . dx$

$= \int x^{7/2} . dx - 3\int x^{5/2} . dx + 2\int x^{1/2} . dx$

$= \frac{x^{9/2}}{9/2} - \frac{3x^{7/2}}{7/2} + \frac{2x^{3/2}}{3/2} + c$

$= \frac{2x^{9/2}}{9} - \frac{6x^{7/2}}{7} + \frac{4x^{3/2}}{3} + c$

$= x^{3/2}\left(\frac{2}{9}x^3 - \frac{6}{7}x^2 + \frac{4}{3}\right) + c$

$= 2x^{3/2}\left(\frac{x^3}{9} - \frac{3}{7}x^2 + \frac{2}{3}\right) + c$

(92) $\int \frac{8}{(2x+3)} . dx$

$= 8\int \frac{1}{2x+3} . dx$

$= 8 \times \frac{\text{Log}(2x+3)}{2} + c$

$= 4 . \text{Log}(2x+3) + c$

cross-check

$\frac{d}{dx} 4 . \text{Log}(2x+3) + c$

$= 4 \times \frac{1}{2x+3} \times 2 + 0$

$= \left(\frac{8}{2x+3}\right)$

(93) $\int \left(\frac{11}{19x-15} + \frac{1}{x} + 3 \right) . dx$

$= \int \frac{11}{19x-15} . dx + \int \frac{1}{x} . dx + \int 3 . dx$

$= 11 \times \frac{\text{Log}(19x-15)}{19} + \text{Log}x + 3x + c$

(94) $\int \left(e^{4x} + e^{-4x} \right) . dx$

$\Longrightarrow \int e^{4x} . dx + \int e^{-4x} . dx$

$= \frac{e^{4x}}{4} + \frac{e^{-4x}}{-4} + c$

$= \left(\frac{e^{4x} - e^{-4x}}{4} \right) + c$

(95) $\int \left(\frac{x^2}{x+1} \right) dx$

$= \int \frac{x^2 - 1 + 1}{x+1} . dx$

$= \int \frac{(x^2-1)}{(x+1)} + \frac{1}{(x+1)} . dx$

$= \int \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{1}{(x+1)} . dx$

$= \int x - 1 + \frac{1}{(x+1)} . dx$

$= \frac{x^2}{2} - x + \text{Log}(x+1) + c$

cross-check

$\frac{d}{dx} \left(\frac{x^2}{2} - x + \text{Log}(x+1) \right)$

$= \frac{1}{2} \times 2x - 1 + \frac{1}{x+1} \times 1$

$= x - 1 + \frac{1}{x+1} + 0$

$= \frac{x(x+1) - (x+1) + 1}{(x+1)}$

$= \frac{x^2 + \cancel{x} - \cancel{x} - \cancel{1} + \cancel{1}}{(x+1)}$

$= \left(\frac{x^2}{x+1} \right)$

(96) $\int \log x \cdot dx$

$= \int (\log x) \times 1 \; dx$

By using Integration by parts

$= \log x \times \int 1 \cdot dx - \int \left[\frac{d}{dx} \log x \times \int 1 \cdot dx\right] \cdot dx$

$= \log x \times x - \int \left(\frac{1}{x} \times x\right) dx$

$= x \cdot \log x - \int 1 \cdot dx$

$= x \cdot \log x - x + c$

$= x (\log x - 1) + c$

Integration by parts

$\int (u \times v) \cdot dx$

$= u \times \int v \cdot dx - \int \left[\frac{du}{dx} \times \int v \cdot dx\right] \cdot dx$

(97) $I = \int \frac{x^3}{(x^2+1)^3} \cdot dx$

$\Longrightarrow$ suppose $\boxed{t = x^2 + 1}$ $\therefore x^2 = t - 1$

$\frac{dt}{dx} = 2x$

$\therefore dx = \frac{dt}{2x}$

$I = \int \frac{x^3}{t^3} \cdot \frac{dt}{2x}$

$I = \frac{1}{2} \int \frac{x^2}{t^3} \cdot dt = \frac{1}{2} \int \left(\frac{t-1}{t^3}\right) \cdot dt$

$= \frac{1}{2} \int \left(\frac{t}{t^3} - \frac{1}{t^3}\right) \cdot dt = \frac{1}{2} \int \left(t^{-2} - t^{-3}\right) \cdot dt$

$$= \frac{1}{2}\left[\int t^{-2}.dt - \int t^{-3}.dt\right]$$

$$= \frac{1}{2}\left[\frac{t^{-1}}{-1} - \frac{t^{-2}}{-2}\right] + c$$

$$= \frac{1}{2}\left(\frac{-1}{t} + \frac{1}{2t^2}\right) + c$$

$$= \frac{-2}{t} + \frac{1}{4t^2} + C$$

$$= \frac{-2}{x^2+1} + \frac{1}{4(x^2+1)^2} + c$$

98 Definite Integrals

$$\boxed{\begin{aligned} &\int f(x).dx = g(x) \\ &\text{then } \int_a^b f(x).dx = g(b) - g(a) \end{aligned}}$$

$$\int_3^5 (x^2+3).dx$$

$$= \left[\frac{x^3}{3} + 3x\right]_3^5 = \left[\frac{5^3}{3} + 3(5)\right] - \left[\frac{3^3}{3} + 3(3)\right]$$

$$= \frac{125}{3} + 15 - \frac{27}{3} - 9$$

$$= \frac{125}{3} + 15 - 9 - 9$$

$$= \frac{125}{3} - 3 = \left(\frac{116}{3}\right)$$

(99) $\int_{8}^{12} \left(\frac{1}{x}\right).dx$

$= \left[\text{Log}\,x\right]_{8}^{12}$

$= \text{Log}\,12 - \text{Log}\,8 = \text{Log}\left(12/8\right) = \text{Log}_e\left(3/2\right)$

(100) $\int_{-2}^{5} x.dx$

$= \left[\frac{x^2}{2}\right]_{-2}^{5}$

$= \frac{5^2}{2} - \frac{(-2)^2}{2} = \frac{25}{2} - \frac{4}{2} = \left(\frac{21}{2}\right)$

(101) $\int_{2}^{5} e^{7x+3}.dx$

$= \left[\frac{e^{7x+3}}{7}\right]_{2}^{5}$

$= \left(\frac{e^{38}}{7} - \frac{e^{17}}{7}\right) = \frac{e^{17}}{7}\left(e^{21} - 1\right)$

$$\boxed{\begin{array}{l} \text{If } \int f(x).dx = g(x) \\ \text{then} \\ \int_{p}^{q} f(x) = g(q) - g(p) \end{array}}$$

(102) $\frac{d}{dx} \text{Log}\left[f(x)\right]$

$= \frac{1}{f(x)} \times f'(x)$

$= \frac{f'(x)}{f(x)}$

$$\boxed{\begin{array}{l} \therefore \int \frac{f'(x)}{f(x)}.dx \\ = \text{Log}\,f(x) + c \end{array}}$$

(103) $$\int \left[\frac{10x+2}{5x^2+2x+3}\right] . dx$$

$$= \int \frac{f'(x)}{f(x)} . dx$$

$f(x) = 5x^2 + 2x + 3$

$f'(x) = 10x + 2$

$$= \text{Log } f(x) + c$$

$$= \text{Log}\left(5x^2+2x+3\right) + c$$

(104) $$\int \frac{18x - 9}{9x^2 - 9x + 35} . dx$$

$$= \text{Log}\left(9x^2-9x+35\right) + c$$

$f(x) = 9x^2 - 9x + 35$

$f'(x) = 18x - 9$

(105) (a) $$\int \left(\frac{1}{x^2 - a^2}\right) dx = \frac{1}{2a} \text{Log}\left|\frac{x-a}{x+a}\right| + c$$

(b) $$\int \left(\frac{1}{a^2 - x^2}\right) . dx = \frac{1}{2a} \text{Log}\left|\frac{a+x}{a-x}\right| + c$$

(c) $$\int \frac{1}{\sqrt{x^2+a^2}} . dx = \text{Log}\left|x + \sqrt{x^2+a^2}\right| + c$$

(d) $$\int \frac{1}{\sqrt{x^2-a^2}} . dx = \text{Log}\left|x + \sqrt{x^2-a^2}\right| + c$$

(e) $$\int e^x \left[f(x) + f'(x)\right] = e^x . f(x) + c$$

(f) $$\int \sqrt{x^2+a^2} . dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2+a^2}\right| + c$$

(g) $$\int \sqrt{x^2-a^2} . dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \text{Log}\left|x + \sqrt{x^2-a^2}\right| + c$$